

GAUGE FIELD THEORY OF HORIZONTAL $SU(2) \times U(1)$ SYMMETRY

- DOUBLET PLUS SINGLET SCHEME -

Ikuo S. Sogami¹

*Maskawa Institute for Science and Culture, Kyoto Sangyo University,
Kyoto, 603-8555, Japan*

Abstract

Gauge field theory of a horizontal symmetry of the group $G_H = SU(2)_H \times U(1)$ is developed so as to generalize the standard model of particle physics. All fermion and scalar fields are assumed to belong to doublets and singlets of the group in high energy regime. Mass matrices with four texture zeros of Dirac and Majorana types are systematically derived. In addition to seven scalar particles, the theory predicts existence of one peculiar vector particle which seems to play important roles in astrophysics and particle physics.

1 Introduction

To generalize the standard model of particle physics, we develop a gauge field theory of a horizontal (H) symmetry of the group $G_H = SU(2)_H \times U(1)$. Above a high energy scale $\bar{\Lambda}$ which is much higher than the electroweak (EW) scale Λ , fundamental fermions, quarks and leptons, are postulated to form doublets and singlets of the group

¹E-mail: sogami@cc.kyoto-su.ac.jp

G_H . Classification of the fundamental fermions into chiral sectors consisting of EW doublets ($f = q, \ell$) and singlets ($f = u, d; \nu, e$) is assumed to hold also in the high energy regime.

Breakdown of the symmetry at the scale $\bar{\Lambda}$ (Λ) necessitates Higgs fields of doublet and singlet of the H symmetry which belong to the singlets (doublets) of the EW symmetry. The doublet and singlet composition of the EW and H symmetries for both of the fermion and scalar fields simplifies the formalism and enables us to reduce the number of Yukawa coupling constants. In this theory, mass matrices with four texture zeros of Dirac and Majorana types are systematically derived, and unphysical modes of bosonic fields are excluded by properly adjusting values of parameters in the Higgs potentials.

For the sake of distinction, we use the symbols $\{\tau_1, \tau_2, \tau_3\}$ and Y for the isospin and hypercharge of the EW symmetry, and the symbols $\{\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3\}$ and \bar{Y} for the “isospin” and “hypercharge” of the H symmetry. The color degrees of freedom are not specified for simplicity. We introduce a symbol \bar{t} to indicate the operation of transposition for degrees of the H symmetry.

2 Doublet and singlet composition

The gauge fields of the EW symmetry, $A_\mu^{(2)j}$ ($j = 1, 2, 3$) and $A_\mu^{(1)}$, interact to the currents of EW-isospin τ_j and hypercharge Y with coupling constants g_2 and g_1 . In contrast, we introduce gauge fields of the H symmetry, $\bar{A}_\mu^{(2)j}$ ($j = 1, 2, 3$) and $\bar{A}_\mu^{(1)}$, which interact vectorially to the currents generated by H-isospin $\bar{\tau}_j$ and H-hypercharge \bar{Y} with coupling constants \bar{g}_2 and \bar{g}_1 .

In high-energy region ($> \bar{\Lambda}$), fundamental fermions in sector $f (= q, u, d; \ell, \nu, e)$ are postulated to belong to the doublet and singlet of the group G_H as follows:

$$\psi_d^f = \bar{t} \begin{pmatrix} \psi_1^f & \psi_2^f \end{pmatrix}, \quad \psi_s^f = \begin{pmatrix} \psi_3^f \end{pmatrix}, \quad (1)$$

whose components are either the EW chiral doublets as

$$\psi_i^q = \begin{pmatrix} \psi_i^u \\ \psi_i^d \end{pmatrix}_L; \quad \psi_i^\ell = \begin{pmatrix} \psi_i^\nu \\ \psi_i^e \end{pmatrix}_L \quad (2)$$

or the EW chiral singlets as

$$(\psi_i^u)_R, \quad (\psi_i^d)_R; \quad (\psi_i^\nu)_R, \quad (\psi_i^e)_R. \quad (3)$$

In the low-energy region ($\leq \Lambda$), the doublet ψ_d^f and the singlet ψ_s^f turn out to constitute, respectively, main components of the first and second generations and the third generation of fundamental fermions.

To properly break the H and EW symmetries, two types of H multiplets of Higgs fields are presumed to exist. For the H symmetry breaking around the high-energy scale $\bar{\Lambda}$, a set of doublet and singlet of Higgs fields are introduced as

$$\phi_d = \bar{t} \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix}, \quad \phi_s = (\phi_3), \quad (4)$$

where complex fields ϕ_1 and ϕ_2 and a real field ϕ_3 belong to EW singlets. These scalar fields do not couple with the fundamental fermion fields in (1) except for the right-handed neutrino fields $\psi_{d,s}^\nu$. It is this character of ϕ_a that protects the fundamental fermion fields from acquiring Dirac masses of the scale $\bar{\Lambda}$. A dual doublet of ϕ_d is defined by $\tilde{\phi}_d = i\bar{\tau}_2 \phi_d^*$.

To form the Yukawa interaction and break its symmetry at the scale Λ , a set of H doublet and singlet consisting of three EW doublets must exist as

$$\varphi_d = \bar{t} \begin{pmatrix} \varphi_1 & \varphi_2 \end{pmatrix} = \bar{t} \left(\begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}, \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix} \right), \quad \varphi_s = \begin{pmatrix} \varphi_3^+ \\ \varphi_3^0 \end{pmatrix}, \quad (5)$$

which, respectively, have dual multiplets $\tilde{\varphi}_d = (i\bar{\tau}_2)(i\tau_2)\varphi_d^*$ and $\tilde{\varphi}_s = i\tau_2\varphi_s^*$.

3 Lagrangian density

The Lagrangian density for the fermion and scalar interactions consists of the Yukawa and Majorana parts. The density of the Yukawa interaction, \mathcal{L}_Y^f , consists of the EW \times H invariants of the multiplets ψ_a and φ_a ($a = d, s$) as follows:

$$\mathcal{L}_Y^f = \mathcal{Y}_1^f \bar{\psi}_d^{f'} \tilde{\varphi}_s \psi_d^f + \mathcal{Y}_2^f \bar{\psi}_d^{f'} \tilde{\varphi}_d \psi_s^f + \mathcal{Y}_3^f \bar{\psi}_s^{f'} \tilde{\varphi}_d i\bar{\tau}_2 \psi_d^f + \mathcal{Y}_4^f \bar{\psi}_s^{f'} \tilde{\varphi}_s \psi_s^f + \text{h.c.} \quad (6)$$

for the EW up-sectors ($f' = q, f = u$) and ($f' = \ell, f = \nu$), and

$$\mathcal{L}_Y^f = \mathcal{Y}_1^f \bar{\psi}_d^{f'} \varphi_s \psi_d^f + \mathcal{Y}_2^f \bar{\psi}_d^{f'} \varphi_d \psi_s^f + \mathcal{Y}_3^f \bar{\psi}_s^{f'} \varphi_d i\bar{\tau}_2 \psi_d^f + \mathcal{Y}_4^f \bar{\psi}_s^{f'} \varphi_s \psi_s^f + \text{h.c.} \quad (7)$$

for the EW down-sectors ($f' = q, f = d$) and ($f' = \ell, f = e$). Four unknown complex coupling constants \mathcal{Y}_{fi} ($i = 1, \dots, 4$) exist in each

sector. The Lagrangian density for the Majorana interaction, \mathcal{L}_M , is given by

$$\mathcal{L}_M = \bar{g} \left(\overline{\psi_d^{\nu c}} \bar{\tau}_2 \phi_d \psi_s^\nu + \overline{\psi_s^{\nu c}} \bar{\tau}_2 \phi_d \psi_d^\nu \right) + \bar{M}_d \overline{\psi_d^{\nu c}} \bar{\tau}_2 \psi_d^\nu + \bar{M}_s \overline{\psi_s^{\nu c}} \bar{\tau}_2 \psi_s^\nu, \quad (8)$$

where $\psi_a^{\nu c}$ are the charge conjugate fields, and \bar{g} and \bar{M}_a ($a = d, s$) are the Majorana coupling constant and masses.

The Lagrangian density for the scalar fields, $\mathcal{L}_{\text{scalar}}$, takes the form

$$\mathcal{L}_{\text{scalar}} = \sum_{a=d,s} (\mathcal{D}^\mu \phi_a)^\dagger (\mathcal{D}_\mu \phi_a) + \sum_{a=d,s} (\mathcal{D}^\mu \varphi_a)^\dagger (\mathcal{D}_\mu \varphi_a) - V_T(\phi, \varphi), \quad (9)$$

where $V_T(\phi, \varphi)$ is the total Higgs potential including all Higgs fields. The covariant derivatives \mathcal{D}_μ for the scalar multiplets ϕ_a and φ_a are given, respectively, by ²

$$\mathcal{D}_\mu \phi_d = \left(\partial_\mu - i\bar{g}_2 \bar{A}_\mu^{(2)j} \frac{1}{2} \bar{\tau}_j - i\bar{g}_1 \bar{A}_\mu^{(1)} \frac{1}{2} \right) \phi_d, \quad (10)$$

$$\mathcal{D}_\mu \phi_s = \partial_\mu \phi_s, \quad (11)$$

$$\mathcal{D}_\mu \varphi_d = \left(\partial_\mu - ig_2 A_\mu^{(2)j} \frac{1}{2} \tau_j - ig_1 A_\mu^{(1)} \frac{1}{2} - i\bar{g}_2 \bar{A}_\mu^{(2)j} \frac{1}{2} \bar{\tau}_j - i\bar{g}_1 \bar{A}_\mu^{(1)} \frac{1}{2} \bar{Y}_{\varphi_d} \right) \varphi_d, \quad (12)$$

and

$$\mathcal{D}_\mu \varphi_s = \left(\partial_\mu - ig_2 A_\mu^{(2)j} \frac{1}{2} \tau_j - ig_1 A_\mu^{(1)} \frac{1}{2} - i\bar{g}_1 \bar{A}_\mu^{(1)} \frac{1}{2} \bar{Y}_{\varphi_s} \right) \varphi_s. \quad (13)$$

The total Higgs potential of the multiplets ϕ_a and φ_a , $V_T(\phi, \varphi)$, can be separated into the sum of three parts as follows:

$$V_T(\phi, \varphi) = V_1(\phi) + V_2(\varphi) + V_3(\varphi; \phi). \quad (14)$$

The potential $V_1(\phi)$ of the self-interactions of the multiplets ϕ_a is given by

$$V_1(\phi) = -\bar{m}_d^2 \phi_d^\dagger \phi_d - \bar{m}_s^2 \phi_s^\dagger \phi_s + \frac{1}{2} \bar{\lambda}_d \left(\phi_d^\dagger \phi_d \right)^2 + \frac{1}{2} \bar{\lambda}_s \phi_s^4 + \bar{\lambda}_{ds} \left(\phi_d^\dagger \phi_d \right) \phi_s^2, \quad (15)$$

where $\bar{\lambda}_s$, $\bar{\lambda}_d$ and $\bar{\lambda}_{ds}$ are positive coupling constants satisfying $\bar{\lambda}_d \bar{\lambda}_s > \bar{\lambda}_{ds}^2$. Using this density, we analyze the breakdown of the H symmetry

²The H-hypercharge of ϕ_d is chosen to be 1 by adjusting the value of the coupling constant \bar{g}_2 .

around the scale $\bar{\Lambda}$. The potential $V_2(\varphi)$ of the self-interactions of the multiplets φ_a is expressed as

$$\begin{aligned}
V_2(\varphi) = & -m_d^2 \varphi_d^\dagger \varphi_d - m_s^2 \varphi_s^\dagger \varphi_s + \frac{1}{2} \lambda_d (\varphi_d^\dagger \varphi_d)^2 + \frac{1}{2} \lambda_{d1} |\varphi_d^\dagger \tilde{\varphi}_d|^2 \\
& + \frac{1}{2} \lambda_{d2} \overline{\text{Tr}} \left(\bar{t} \varphi_d^\dagger \bar{t} \varphi_d \right)^2 + \frac{1}{2} \lambda_{d3} \overline{\text{Tr}} \left(\bar{t} \varphi_d^\dagger \bar{t} \varphi_d \bar{t} \tilde{\varphi}_d^\dagger \bar{t} \tilde{\varphi}_d \right) + \frac{1}{2} \lambda_s (\varphi_s^\dagger \varphi_s)^2 \\
& + \lambda_{ds} \left(\varphi_d^\dagger \varphi_d \right) \left(\varphi_s^\dagger \varphi_s \right) + \lambda_{ds1} |\varphi_d^\dagger \varphi_s|^2 + \lambda_{ds2} |\varphi_d^\dagger \tilde{\varphi}_s|^2, \quad (16)
\end{aligned}$$

where $\overline{\text{Tr}}$ means to take the trace operation with respect to the H-degrees of freedom. For the potential of mutual interactions between the multiplets φ_a and ϕ_a , we obtain

$$\begin{aligned}
V_3(\varphi; \phi) = & \dot{\lambda}_1 (\phi_d^\dagger \phi_d) (\varphi_d^\dagger \varphi_d) + \dot{\lambda}_2 \phi_s^2 (\varphi_d^\dagger \varphi_d) + \dot{\lambda}_3 (\phi_d^\dagger \phi_d) (\varphi_s^\dagger \varphi_s) \\
& + \dot{\lambda}_4 \phi_s^2 (\varphi_s^\dagger \varphi_s) + \dot{\lambda}_5 |\phi_d^\dagger \varphi_d|^2 + \dot{\lambda}_6 |\phi_d^\dagger \varphi_d|^2. \quad (17)
\end{aligned}$$

4 Symmetry breakdown at high-energy scale $\bar{\Lambda}$

In the broken phase of the H symmetry around and below the scale $\bar{\Lambda}$, the doublet and singlet, ϕ_d and ϕ_s , are decomposed into the following forms:

$$\phi_d = \bar{t} \begin{pmatrix} 0, & \bar{v}_d + \frac{1}{\sqrt{2}} \xi_d \end{pmatrix}, \quad \phi_s = \bar{v}_s + \frac{1}{\sqrt{2}} \xi_s, \quad (18)$$

where \bar{v}_d and \bar{v}_s are vacuum expectation values (VEVs), and ξ_d and ξ_s are real component scalar fields. Up to the second order, the potential $V_1(\phi)$ takes the form

$$V_1(\phi) = V_1(\bar{v}) + \bar{\lambda}_d \bar{v}_d^2 \xi_d^2 + \bar{\lambda}_s \bar{v}_s^2 \xi_s^2 + 2\bar{\lambda}_{ds} \bar{v}_d \bar{v}_s \xi_d \xi_s = V_1(\bar{v}) + \frac{1}{2} m_{\xi_1}^2 \xi_1^2 + \frac{1}{2} m_{\xi_2}^2 \xi_2^2, \quad (19)$$

where new real fields ξ_i ($i = 1, 2$) are introduced by

$$\xi_d = \cos \bar{\theta} \xi_1 - \sin \bar{\theta} \xi_2, \quad \xi_s = \sin \bar{\theta} \xi_1 + \cos \bar{\theta} \xi_2. \quad (20)$$

The mixing angle $\bar{\theta}$ is subject to

$$\tan 2\bar{\theta} = \frac{2\bar{\lambda}_{ds} \bar{v}_d \bar{v}_s}{\lambda_d \bar{v}_d^2 - \lambda_s \bar{v}_s^2} \quad (21)$$

and the mass of the field ξ_i is obtained by

$$m_{\xi_i}^2 = \bar{\lambda}_d \bar{v}_d^2 + \bar{\lambda}_s \bar{v}_s^2 + (-1)^i \sqrt{(\bar{\lambda}_d \bar{v}_d^2 - \bar{\lambda}_s \bar{v}_s^2)^2 + 4(\bar{\lambda}_{ds} \bar{v}_d \bar{v}_s)^2}. \quad (22)$$

The symmetry breaking at the scale $\bar{\Lambda}$ metamorphoses the gauge fields $\bar{A}_\mu^{(2)j}$ and $\bar{A}_\mu^{(1)}$ into new fields. Estimation of the action of the covariant derivative on the scalar doublet at the stationary state with the VEV \bar{v}_d results in

$$(\mathcal{D}_\mu \langle \phi_d \rangle)^\dagger (\mathcal{D}^\mu \langle \phi_d \rangle) = M_W^2 \bar{W}_\mu \bar{W}^\mu + \frac{1}{2} M_Z^2 \bar{Z}_\mu \bar{Z}^\mu, \quad (23)$$

where \bar{W}_μ is the complex vector field

$$\bar{W}_\mu = \frac{\bar{A}_\mu^{(2)1} - i \bar{A}_\mu^{(2)2}}{\sqrt{2}} \quad (24)$$

with the mass $M_W^2 = \frac{1}{2} \bar{g}^2 \bar{v}^2$, and \bar{Z}_μ is the neutral vector field

$$\bar{Z}_\mu = \frac{\bar{g}_2 \bar{A}_\mu^{(2)3} - \bar{g}_1 \bar{A}_\mu^{(1)}}{\sqrt{\bar{g}_2^2 + \bar{g}_1^2}} = \bar{A}_\mu^{(2)3} \cos \vartheta - \bar{A}_\mu^{(1)} \sin \vartheta \quad (25)$$

carrying the mass

$$M_Z^2 = \frac{1}{2} (\bar{g}_2 + \bar{g}_1^2) \bar{v}_d^2 = \frac{M_W^2}{\cos^2 \vartheta}. \quad (26)$$

There exists another vector field X_μ , being orthogonal to \bar{Z}_μ , with the configuration

$$X_\mu = \frac{\bar{g}_1 \bar{A}_\mu^{(2)3} + \bar{g}_2 \bar{A}_\mu^{(1)}}{\sqrt{\bar{g}_2^2 + \bar{g}_1^2}} = \bar{A}_\mu^{(2)3} \sin \vartheta + \bar{A}_\mu^{(1)} \cos \vartheta, \quad (27)$$

which remains massless down to the scale Λ and makes gauge interaction to the vector currents of a new charge $\bar{Q} = \frac{1}{2} \bar{\tau}_3 + \frac{1}{2} \bar{Y}$ of the H symmetry with the unit of strength, \bar{e} , defined by

$$\bar{e} = \bar{g}_2 \sin \vartheta = \bar{g}_1 \cos \vartheta. \quad (28)$$

Substitution of the decompositions of ϕ_a in (18) into (8) leads to the effective Lagrangian density of neutrino species as

$$\mathcal{L}_M \rightarrow \mathcal{L}_M^M = \bar{\Psi}_L^{\nu c} \mathcal{M}_\nu \Psi_R^\nu + \dots, \quad (29)$$

where $\Psi_{L,R}^\nu$ are chiral neutrino triplets in the interaction mode, $\bar{\mathcal{M}}_\nu$ is the Majorana mass matrix

$$\bar{\mathcal{M}}_\nu = \begin{pmatrix} 0 & -i\bar{M}_d & -i\bar{g}\bar{v}_d \\ i\bar{M}_d & 0 & 0 \\ i\bar{g}\bar{v}_d & 0 & \bar{M}_s \end{pmatrix}, \quad (30)$$

and the ellipsis means interactions between the neutrinos and scalar fields ξ_i .

5 Symmetry breakdown at low-energy scale Λ

To go down to the low-energy region around the scale Λ , the effects of the renormalization group must properly be taken into account for all of the physical quantities. In particular, all coupling constants run down to the values at the scale Λ . For the sake of simplicity, the same symbols are used here for the quantities including all these effects.

In the broken phase of EW symmetry around the scale Λ , the multiplets φ_a are postulated to take the forms

$$\varphi_d = \bar{t} \left(\begin{pmatrix} \zeta_1^+ \\ \zeta_1^0 \end{pmatrix}, \begin{pmatrix} \zeta_2^+ \\ v_d + \frac{1}{\sqrt{2}}\eta_d \end{pmatrix} \right), \quad \varphi_s = \begin{pmatrix} 0 \\ v_s + \frac{1}{\sqrt{2}}\eta_s \end{pmatrix} \quad (31)$$

with VEVs v_d and v_s , where ζ_1^+ , ζ_2^+ and ζ_1^0 are complex component fields, and η_d and η_s are real component fields. To examine the dynamics around the scale Λ , it is necessary to examine the sum of the potential $V_2(\varphi)$ and also the potential $V_3(\varphi, \bar{v})$ which reflects the influence of the H symmetry breakdown at the high-energy scale $\bar{\Lambda}$. We obtain the stationary conditions as follows:

$$\begin{aligned} (\lambda_d + \lambda_{d2})v_d^2 + (\lambda_{ds} + \lambda_{ds1})v_s^2 &= m_d'^2 \equiv m_d^2 - \dot{\lambda}_1\bar{v}_d^2 - \dot{\lambda}_2\bar{v}_s^2 - \dot{\lambda}_6\bar{v}_d^2, \\ (\lambda_{ds} + \lambda_{ds1})v_d^2 + \lambda_s v_s^2 &= m_s'^2 \equiv m_s^2 - \dot{\lambda}_3\bar{v}_d^2 - \dot{\lambda}_4\bar{v}_s^2. \end{aligned} \quad (32)$$

Accordingly, for the phase transition to take place ($v_d, v_s \neq 0$), reduced quantities $m_d'^2$ and $m_s'^2$ must be positive.

Around the stationary point, the sum of the Higgs potential is decomposed with respect to the component scalar fields, up to the

second order, as

$$\begin{aligned}
V_2(\varphi) + V_3(\varphi; \bar{v}_d) &= V_2(v) + V_3(v; \bar{v}_d) \\
&+ m_{\zeta_1^+}^2 |\zeta_1^+|^2 + m_{\zeta_2^+}^2 |\zeta_2^+|^2 + m_{\zeta_1^0}^2 |\zeta_1^0|^2 + \frac{1}{2} m_1^2 \eta_1^2 + \frac{1}{2} m_2^2 \eta_2^2 \cdots,
\end{aligned} \tag{33}$$

where η_i ($i = 1, 2$) are new real fields introduced by

$$\eta_d = \cos \theta \eta_1 - \sin \theta \eta_2, \quad \eta_s = \sin \theta \eta_1 + \cos \theta \eta_2. \tag{34}$$

The masses of three complex fields ζ_1^+ , ζ_2^+ and ζ_1^0 are calculated to be

$$\begin{aligned}
m_{\zeta_1^+}^2 &= (2\lambda_{d1} - \lambda_{d2} + \lambda_{d3})v_d^2 + m_{\zeta_2^+}^2 + m_{\zeta_1^0}^2, \\
m_{\zeta_2^+}^2 &= (\lambda_{ds2} - \lambda_{ds1})v_s^2, \\
m_{\zeta_1^0}^2 &= (\dot{\lambda}_5 - \dot{\lambda}_6)\bar{v}_d^2.
\end{aligned} \tag{35}$$

The two real fields η_i ($i = 1, 2$) have the masses as

$$m_{\eta_i}^2 = D + S + (-1)^i \sqrt{(D - S)^2 + 4(\lambda_{ds} + \lambda_{ds1})^2 v_d^2 v_s^2}, \tag{36}$$

and the mixing angle $\bar{\theta}$ is subjects to

$$\tan 2\theta = \frac{2(\lambda_{ds} + \lambda_{ds1})v_d v_s}{D - S}, \tag{37}$$

where the abbreviations

$$D = (\lambda_d + \lambda_{d2})v_d^2 - \frac{1}{2}(\lambda_{ds} + \lambda_{ds1})v_s^2, \quad S = \lambda_s v_s^2 \tag{38}$$

are used.

Results in (35), (36) and (38) demonstrate that the Higgs coupling constants must satisfy inequality relations so that all complex and real scalar fields are in physical modes. For example, the inequality relations $\lambda_{ds2} > \lambda_{ds1}$ and $\dot{\lambda}_5 > \dot{\lambda}_6$ must hold to make the masses of the fields ζ_2^+ and ζ_1^0 positive-definite. From (36), it is shown that the real field η_i with lighter mass must be identified with the so-called Higgs particle. Note that stringent conditions on the flavor changing neutral (charged) currents give strong restrictions on the Higgs coupling constants.

The symmetry breaking at the scale Λ changes the gauge fields $A_\mu^{(2)j}$, $A_\mu^{(1)}$ and X_μ into massive vector fields by transferring the four

degrees of freedom of the scalar multiplets φ_a . To determine configurations of the vector fields, we calculate the action of the covariant derivative on the scalar multiplets φ_d and φ_s at their stationary state with the VEVs of v_d and v_s obtaining

$$\begin{aligned} \sum_{a=d,s} (\mathcal{D}_\mu \langle \phi_a \rangle)^\dagger (\mathcal{D}^\mu \langle \phi_a \rangle) &= \frac{1}{2} g_2^2 (v_d^2 + v_s^2) W_\mu W^\mu \\ &+ \frac{1}{4} v_d^2 \left[\frac{g_2}{\cos \theta_W} Z_\mu + \bar{e}(1 - \bar{Y}_{\varphi_d}) X_\mu \right] \left[\frac{g_2}{\cos \theta_W} Z^\mu + \bar{e}(1 - \bar{Y}_{\varphi_d}) X^\mu \right] \\ &+ \frac{1}{4} v_s^2 \left[\frac{g_2}{\cos \theta_W} Z_\mu - \bar{e} \bar{Y}_{\varphi_s} X_\mu \right] \left[\frac{g_2}{\cos \theta_W} Z^\mu - \bar{e} \bar{Y}_{\varphi_s} X^\mu \right] + \dots, \end{aligned} \quad (39)$$

where the charged vector field

$$W_\mu = \frac{A_\mu^{(2)1} - i A_\mu^{(2)2}}{\sqrt{2}}, \quad (40)$$

and the neutral vector field

$$Z_\mu = \frac{g_2 A_\mu^{(2)3} - g_1 A_\mu^{(1)}}{\sqrt{g_2^2 + g_1^2}} = A_\mu^{(2)3} \cos \theta_W - A_\mu^{(1)} \sin \theta_W \quad (41)$$

and

$$A_\mu = \frac{g_1 A_\mu^{(2)3} + g_2 A_\mu^{(1)}}{\sqrt{g_2^2 + g_1^2}} = A_\mu^{(2)3} \sin \theta_W + A_\mu^{(1)} \cos \theta_W \quad (42)$$

are introduced, in exactly the same way with the Weinberg-Salam theory by using the Weinberg angle θ_W related to the unit e of electromagnetic interaction as

$$g_2 \sin \theta_W = g_1 \cos \theta_W = e. \quad (43)$$

The ellipsis in (39) implies mass corrections to the super-massive vector fields \bar{W}_μ and \bar{Z}_μ , and mixing interactions of the field \bar{Z}_μ with the fields Z_μ and X_μ .

The charged vector field W_μ possesses the mass

$$M_W^2 = \frac{1}{2} g_2^2 (v_d^2 + v_s^2). \quad (44)$$

The quadratic part of the neutral fields Z_μ and X_μ in (39) is readily diagonalized provided that

$$(1 - \bar{Y}_{\varphi_d}) v_d^2 = \bar{Y}_{\varphi_s} v_s^2. \quad (45)$$

Under this condition, the masses of the fields Z_μ and X_μ are determined to be

$$M_Z^2 = \frac{1}{2} \frac{g_2^2}{\cos^2 \theta_W} (v_d^2 + v_s^2) = \frac{M_W^2}{\cos^2 \theta_W} \quad (46)$$

and

$$M_X^2 = \frac{1}{2} \frac{\bar{e}^2 v_s^2}{v_d^2} (v_d^2 + v_s^2) \bar{Y}_{\varphi_s}^2 = \frac{\bar{e}^2 v_s^2}{g_2^2 v_d^2} \bar{Y}_{\varphi_s}^2 M_W^2. \quad (47)$$

The mass relation in (46) proves that the firmly-established experimental criterion for the standard model, $\rho = M_W^2/M_Z^2 \cos^2 \theta_W = 1$, holds also in the present theory.

Through the breakdown of EW symmetry at the scale Λ , the fermion fields acquire Dirac type masses. Substitution of the decomposition of φ_a in (31) into (6) and (7) leads to the effective Lagrangian density for the fermion fields in the low-energy region as

$$\mathcal{L}_Y \rightarrow \mathcal{L}_M^Y = \sum_{f=u,d,\nu,e} \bar{\Psi}_L^f \mathcal{M}_f \Psi_R^f + \text{h.c.} + \cdots, \quad (48)$$

where $\Psi_{L,R}^f$ are the chiral triplets of f -sector in the interaction mode, and \mathcal{M}_f are the Dirac mass matrices. The ellipsis stands for the interactions of fermion and scalar fields. For the up-sectors ($f = u, \nu$) of EW symmetry, we deduce the Dirac mass matrices as follows:

$$\mathcal{M}_f = \begin{pmatrix} \mathcal{Y}_1^u v_s & 0 & \mathcal{Y}_2^u v_d \\ 0 & \mathcal{Y}_1^u v_s & 0 \\ 0 & \mathcal{Y}_3^u v_d & \mathcal{Y}_4^u v_s \end{pmatrix}. \quad (49)$$

Likewise, for the down-sectors ($f = d, e$) of EW symmetry, we obtain

$$\mathcal{M}_f = \begin{pmatrix} \mathcal{Y}_1^d v_s & 0 & 0 \\ 0 & \mathcal{Y}_1^d v_s & \mathcal{Y}_2^d v_d \\ -\mathcal{Y}_3^d v_d & 0 & \mathcal{Y}_4^d v_s \end{pmatrix}. \quad (50)$$

Both matrices which have four texture zeros are characterized by four independent parameters. It should be recognized that all the parameters except for two can be set to be real by adjusting phases of the doublets and singlets of the fermion and scalar fields, ψ_a^f and φ_a , in the Yukawa interactions in (6) and (7).

6 Discussion

Thanks to the unique construction of the present theory in which all the fermion and scalar fields are presumed to belong to the doublet and singlet representations of the H and EW symmetries, we have succeeded systematically to obtain simple mass matrices with four texture zeros as in (30), (49) and (50). Accordingly, it is tempting to inquire why there exists such a kind of duality that the symmetry $SU(2) \times U(1)$ holds both in the H and EW degrees of freedom.

For the quark sector, the eigenvalue problems for $\mathcal{M}_f \mathcal{M}_f^\dagger$ ($f = u, d$) which have ten adjustable parameters provide sufficient information on the mass spectra and flavor mixing matrix. Smallness of neutrino masses is usually explained by the seesaw mechanism in which the inverse of the Majorana mass matrix with large components works to suppress the Dirac matrix. Remark that the determinant of the Majorana mass matrix in (30) is calculated to be $|\tilde{\mathcal{M}}_\nu| = -\tilde{M}_d^2 \tilde{M}_s$. Therefore, the seesaw suppression occurs exclusively by the Majorana masses \tilde{M}_a independently of the VEV \bar{v}_d . This observation reveals that the present scheme might be reinterpreted to have three energy scales, $\tilde{M}_a \gg \tilde{\Lambda} \gg \Lambda$, rather than two scales, $\tilde{\Lambda} \gg \Lambda$.

In this theory, values of the coupling constants in the Higgs potential must be tuned so that the symmetry breakdowns properly take place and all bosonic fields acquire positive masses. Furthermore, those coupling constants must obey much stronger conditions so that the stringent experimental criteria of the flavor changing neutral (charged) currents are fulfilled.

In addition to the seven scalar particles related to the fields $\xi_1, \xi_2; \zeta_1^+, \zeta_2^+, \zeta_1^0, \eta_1$ and η_2 , the theory predicts existence of one peculiar particle described by the vector field X_μ interacting with current of the charge $\bar{Q} = \frac{1}{2}\bar{\tau}_3 + \bar{Y}$ of the horizontal symmetry. Search for the signals of these particles are expected as possible targets for the LHC experiment. Through massless and massive stages, the exotic field X_μ seems to play important roles in astrophysics and particle physics.